**Topics: Normal distribution, Functions of Random Variables**

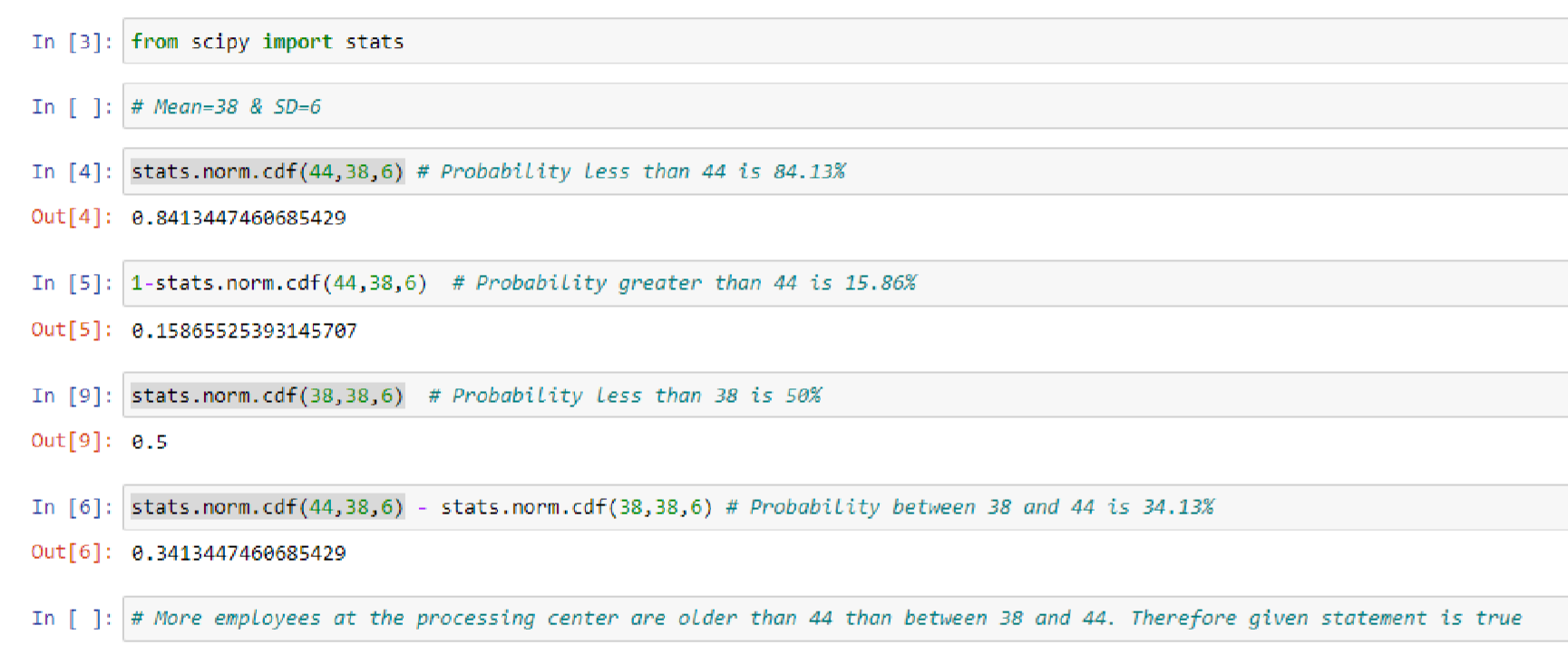
1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

* Ans : (B) 0.2676

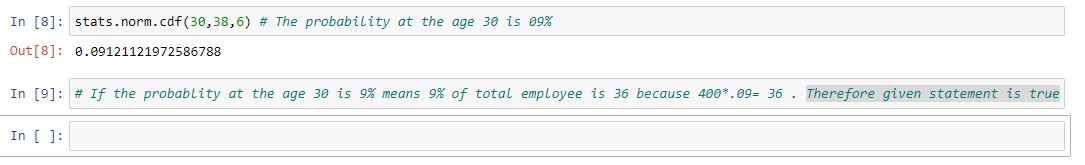


1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean = 38 and Standard deviation =6. For each statement below, please specify True/False. If false, briefly explain why.
   * 1. More employees at the processing center are older than 44 than between 38 and 44.

* True



* + 1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
* True



1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

* As we know that if X  N(µ1, σ1^2 ), and Y  N(µ2, σ2^2 ) are two independent random variables then X + Y  N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y  N(µ1 − µ2, σ1^2 + σ2^2 ).
* Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z  N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).
* Therefore in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

* + 1. 90.5, 105.9
    2. 80.2, 119.8
    3. 22, 78
    4. 48.5, 151.5
    5. 90.1, 109.9
* D – 48.5, 151.5
* Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.
* The Probability of getting value between a and b should be 0.99.
* So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).
* The Probability towards left from a = -0.005 (ie. 0.01/2).
* The Probability towards right from b = +0.005 (ie. 0.01/2).
* So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.
* By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
2. Specify the 5th percentile of profit (in Rupees) for the company
3. Which of the two divisions has a larger probability of making a loss in a given year?

